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Optical spatial solitons: historical overview and recent advances

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Abstract

Solitons, nonlinear self-trapped wavepackets, have been extensively studied in many and diverse branches of physics such as optics, plasmas, condensed matter physics, fluid mechanics, particle physics and even astrophysics. Interestingly, over the past two decades, the field of solitons and related nonlinear phenomena has been substantially advanced and enriched by research and discoveries in nonlinear optics. While optical solitons have been vigorously investigated in both spatial and temporal domains, it is now fair to say that much soliton research has been mainly driven by the work on optical spatial solitons. This is partly due to the fact that although temporal solitons as realized in fiber optic systems are fundamentally one-dimensional entities, the high dimensionality associated with their spatial counterparts has opened up altogether new scientific possibilities in soliton research. Another reason is related to the response time of the nonlinearity. Unlike temporal optical solitons, spatial solitons have been realized by employing a variety of noninstantaneous nonlinearities, ranging from the nonlinearities in photorefractive materials and liquid crystals to the nonlinearities mediated by the thermal effect, thermophoresis and the gradient force in colloidal suspensions. Such a diversity of nonlinear effects has given rise to numerous soliton phenomena that could otherwise not be envisioned, because for decades scientists were of the mindset that solitons must strictly be the exact solutions of the cubic nonlinear Schrödinger equation as established for ideal Kerr nonlinear media. As such, the discoveries of optical spatial solitons in different systems and associated new phenomena have stimulated broad interest in soliton research. In particular, the study of incoherent solitons and discrete spatial solitons in optical periodic media not only led to advances in our understanding of fundamental processes in nonlinear optics and photonics, but also had a very important impact on a variety of other disciplines in nonlinear science. In this paper, we provide a brief overview of optical spatial solitons. This review will cover a variety of issues pertaining to self-trapped waves supported by different types of nonlinearities, as well as various families of spatial solitons such as optical lattice solitons and surface solitons. Recent developments in the area of optical spatial solitons, such as 3D light bullets, subwavelength solitons, self-trapping in soft condensed matter and spatial solitons in systems with parity–time symmetry will also be discussed briefly.

(Some figures may appear in colour only in the online journal)

This article was invited by P Chaikin.

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1. Introduction to optical spatial solitons

Optical spatial solitons (self-trapped optical beams) have been the subject of intense research in nonlinear optics, particularly over the past two decades. In this section, we give a brief introduction to the history and properties of optical spatial solitons. Other aspects associated with some of their fascinating characteristics will also be discussed.

1.1. Optical spatial solitons

Solitons are localized wave entities that can propagate in nonlinear media while maintaining a constant shape. They ubiquitously occur in many branches of physics including hydrodynamics, plasma physics, nonlinear optics and Bose–Einstein condensates. In optics, an optical wavepacket (a pulse or a beam) has a natural tendency to spread as it propagates in a medium, either due to chromatic dispersion or as a result of spatial diffraction. Most often, when this natural broadening is eliminated through a nonlinear process, a stable self-localized wavepacket forms. Such a self-trapped wavepacket, whether in time or space or both, is known as an optical soliton. *Optical spatial solitons* are self-trapped optical beams that propagate in a nonlinear medium without diffraction, i.e. their beam diameter remains invariant during propagation [1, 2]. Intuitively, a spatial soliton represents an exact balance between diffraction and nonlinearly induced self-lensing or self-focusing effects. It can also be viewed as an optical beam that induces a waveguide that, in turn, guides itself throughout propagation as if it were confined in an optical fiber. A top-view photograph showing the sharp transition between nonlinear self-trapping and linear diffraction for an optical beam propagating through a photorefractive crystal is displayed in figure 1, where the self-trapped optical beam represents a typical example of a two-dimensional (2D) optical spatial soliton. Likewise, in a way fully analogous to the spatial case, *optical temporal solitons* are non-spreading optical pulses formed when the group velocity dispersion is totally counteracted by nonlinear self-phase modulation effects [3, 4].

Over the years, there has been some debate on the use of the word ‘soliton’ in actual physical settings. Historically, the notion ‘soliton’ emerged from mathematics and was strictly reserved for optical self-trapped wavepackets that happen to obey integrable nonlinear partial differential equations. In

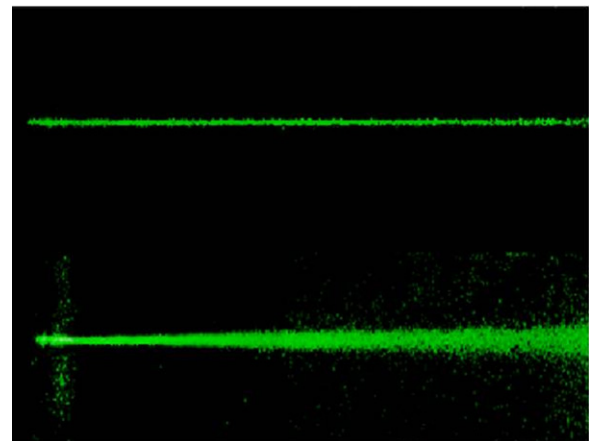


Figure 1. Experimental demonstration of an optical spatial soliton propagating through a 5 mm long nonlinear photorefractive crystal. Top: side-view of the soliton beam from scattered light; bottom: normal diffraction of the same beam when the nonlinearity is ‘turned off’ [28, 29]. Reproduced with permission from [29]. Copyright 1996 The Optical Society.

nonlinear optics, the so-called nonlinear Schrödinger equation (NLSE) represents such an example. The one-dimensional (1D) NLSE that governs wave propagation in ideal Kerr nonlinear media can be fully solved (or integrated) using the ‘inverse scattering theory’ [5, 6], leading to soliton solutions, which remain invariant even after a collision event. In reality, however, most nonlinear physical systems of importance involve non-Kerr or other types of nonlinearity and hence are described by non-integrable evolution equations. Initially, self-trapped entities in non-integrable systems were referred to as ‘solitary waves’ in order to distinguish the interaction and collision properties of these waves from those associated with perfect ‘solitons’ in integrable systems. Yet given that in many cases ‘solitary waves’ display behavior akin to actual solitons, this nomenclature distinction is no longer used in today’s literature. Thus, all self-trapped beams are now loosely called optical spatial solitons, regardless of the actual nonlinearity used to form them.

1.2. How did optical spatial solitons emerge into optics?

The idea that an optical beam can induce a waveguide and guide itself in it was first suggested by Askar’yan as early as

1962 [7]. A few years later, optical beam self-focusing was first observed in materials with *Kerr* nonlinearities [8]. In order to investigate this effect, the wave equation in nonlinear *Kerr* media was analyzed in both one and two transverse dimensions, and spatial self-trapping of optical beams was proposed by Chiao *et al* [9]. However, soon after, Kelley found that the 2D soliton solutions of the NLSE undergo catastrophic collapse and are thus unstable [10]. Even 1D solitons are also unstable in a 3D bulk nonlinear medium, since they can break up into multiple filaments due to transverse instabilities. Thus, stable spatial solitons in *Kerr* media can only exist in configurations where one of the two transverse dimensions is redundant, i.e. where diffraction is arrested in one dimension by some other means such as using a slab waveguide. A few years after Kelley's paper, Dawes and Marburger found numerically that *saturable* nonlinearities are capable of 'arresting' this catastrophic collapse and can lead to stable 2D spatial solitons [11]. However, these ideas have been largely ignored for 20 years. It was not until in the early 1990s, with the discovery of photorefractive solitons and the subsequent observation of quadratic solitons, that saturable nonlinearities for the stable formation of 2D solitons began to be explored in experimental systems. Essentially, the very idea of a 'saturable nonlinearity', associated with materials in which the magnitude of the nonlinear index change has an upper bound (saturates with increasing intensity), turned out to be the key to many of the new families of solitons discovered in the 1990s.

The first experiment on optical spatial solitons was reported in 1974 by Ashkin and Bjorkholm [12]. They used a 2D circularly symmetric beam in a nonlinear medium (a cell filled with sodium vapor) and observed self-trapping at higher powers. This is the classical signature of a spatial soliton. The nonlinearity encountered in the sodium vapor system was not of the classical *Kerr*, but rather of the saturable self-focusing type that exists near an electronic resonance in a two-level system. The saturable nature of this latter nonlinearity arises because intense fields tend to reduce the population difference between the two energy levels, and hence no additional change in the refractive index occurs with any further increase in intensity. This saturable nature of the nonlinearity is essential because, as predicted by theory, 2D spatial solitons are only stable under saturable nonlinear conditions. However, at the same time, this atomic system also exhibits considerable loss, because it occurs at the close proximity of an atomic resonance. The loss inevitably limits the propagation distance and the ability to observe soliton dynamics. One way or the other, Ashkin and Bjorkholm did not continue to pursue research on solitons, and the field was deserted without experiments for another decade. The next experiment on spatial solitons was carried out in 1985, when self-trapping of an optical beam in a 1D planar waveguide filled with liquid CS₂ was observed (as the first 1D *Kerr* spatial soliton) by Barthelemy *et al* [13]. This soliton experiment opened the way to subsequent 1D soliton demonstrations in a variety of materials displaying a *Kerr*-type nonlinearity, including glass, semiconductors and polymers [14–17].

Despite these efforts, most spatial soliton experiments were carried out in 1D configurations using *Kerr*-type

nonlinear media, partly because exact analytical solutions could only be found for the 1D *Kerr* nonlinear NLSE, and partly because researchers were under the (wrong) impression that other kinds of nonlinear mechanisms would not be able to support the formation of stable solitons. In fact, apart from Ashkin and Bjorkholm's first soliton experiment, it was never fully possible to generate a 2D circular soliton propagating as a long needle of light without diffraction. This is mainly for the following two reasons. The first reason is the requirement for exceedingly high powers needed to observe spatial solitons through the optical *Kerr* effect. This is because *Kerr*-type nonlinearities are of electronic origin and are therefore very weak. The second reason, as mentioned above, is even more fundamental. For non-saturable *Kerr* nonlinearities, light-induced lensing tends to become stronger and stronger as the beam intensity increases, thus causing catastrophic self-focusing and breakup of the beam. As we shall elaborate on later, the first successful demonstrations of new classes of spatial solitons that do not necessarily rely on *Kerr* nonlinearities were crucial for the development of this field. In this respect, there were photorefractive solitons that led the way, followed by quadratic solitons, nematicons in nonlinear liquid crystals and spatial solitons in nonlocal nonlinear media.

1.3. Why are spatial solitons so interesting?

Optical spatial solitons are not only interesting in themselves, but also because they exhibit many fascinating features such as particle-like interactions during collisions. Apart from fundamental aspects, spatial solitons have also been suggested for a variety of applications, including for example waveguiding and beam-splitting, optical interconnects, frequency conversion, image transmission, gateless computing and soliton-based navigation.

The equivalence between solitons and particles was suggested in 1965 [18] when soliton collisions were first investigated. It was found that any collision between solitons involves 'forces': solitons interact very much like real particles, exerting attraction and repulsion forces on one another [19]. Such particle-like collisions were subsequently demonstrated with 1D *Kerr* spatial solitons in glass waveguides by Aitchison and co-workers [20, 21]. They showed that two in-phase *Kerr* solitons attract whereas two out-of-phase solitons repel each other. More complex soliton interactions involving coherent four-wave mixing were reported by Shalaby *et al* [22]. These two experiments demonstrated some of the basic collision properties of conventional *Kerr* solitons. Under standard launching conditions, soliton collisions occurring in the 1D domain are fully elastic and hence the number of solitons is always conserved. Yet energy exchange between solitons, in addition to attraction and repulsion, could also occur under different initial conditions. Soon after the discovery of photorefractive solitons [23–31], this situation drastically changed since a much broader class of collision experiments could be accessed experimentally. For example, the inelastic collision between two spatial solitons originating from a saturable nonlinearity was found to lead to soliton fusion or fission, with particle-like annihilation or birth of new solitons (see, e.g., [1, 2] for a detailed review).

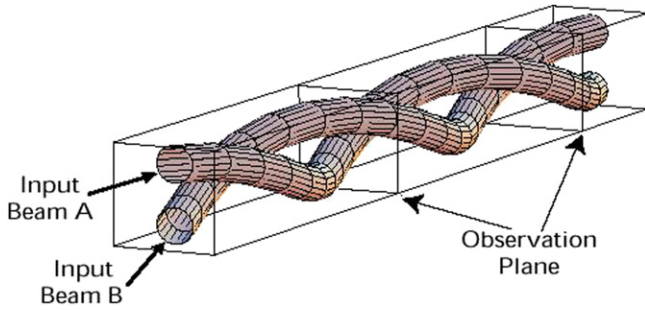


Figure 2. An illustration of spiraling collision of interacting spatial solitons as demonstrated in the experiment with photorefractive solitons [31]. The arrows at input indicate the initial launching directions of the two soliton beams. Reproduced with permission from [31]. Copyright 1997 American Physical Society.

Another interesting aspect of this class of solitons is that spatial solitons are by nature (2+1)D entities (i.e. the two transverse dimensions plus one propagation coordinate), whereas temporal solitons are typically associated with a (1+1)D space–time evolution. With the ability to generate stable 2D solitons, a variety of new phenomena became possible; among them are 3D interactions between solitons, vortex solitons and angular momentum effects, and rotating dipole solitons. The fact that the spatial domain involves a higher dimensionality leads to a host of interesting phenomena and processes, which cannot be demonstrated with 1D Kerr spatial solitons and have no analog whatsoever in the temporal case. Perhaps one of the most intriguing examples is the 3D collision of interacting 2D spatial solitons spiraling around each other in a helical DNA-like trajectory, as illustrated in figure 2 [31].

Over the past two decades, the interest in optical spatial solitons has surged. As in any other research area, there have been certain key ideas and experiments that have defined this field and the directions it has taken. In this review we discuss a selection of such papers and their significance in the development of this field. Given that the activity that has taken place in this area is already enormous, this overview can be by no means complete.

2. Spatial solitons supported by different types of nonlinearities

Before the early 1990s, nearly all experimental demonstrations of spatial solitons employed Kerr or Kerr-like nonlinearities. The discovery of new classes of solitons in material systems different from those with standard Kerr nonlinearities opened up new possibilities in this area. The two major new classes of solitons discovered, at least in terms of nonlinear mechanisms, are photorefractive solitons and quadratic solitons. In what follows, we briefly discuss these two types of spatial solitons.

2.1. Photorefractive solitons

The discovery of photorefractive solitons [23–26] was, for many reasons, a radical turning point in the development of the field of spatial solitons. These solitons are formed

due to multiple physical effects associated with the nonlinear *photorefractive effect*, by which an index change can be established in a photorefractive material through a non-uniform light illumination. In essence, this process involves the absorption of light and subsequent charge generation, the motion of charge under the influence of electric fields, and the consequent establishment of local space-charge fields, which lead to an index change Δn via the electro-optic effect. The nonlinear response is typically non-local (due to the charge migration over macroscopic distances) and non-instantaneous (due to the dielectric relaxation and charge recombination). As such, the photorefractive nonlinearity is also inherently saturable, and can occur even with low-power nonuniform illumination. Because of these unique features of the photorefractive nonlinearity, photorefractive materials have provided a convenient and ideal platform for the observation of a variety of spatial soliton phenomena, including 2D soliton interaction and spiraling and, subsequently, incoherent solitons and discrete solitons.

Photorefractive solitons were first predicated and observed in the quasi-steady-state regime in 1992 by Segev and colleagues [23, 24]. Shortly thereafter, steady-state photorefractive self-focusing was reported [27]. The formation of the so-called steady-state photorefractive ‘screening solitons’ was soon after predicted by Segev *et al* [25] and Christodoulides and Carvalho [26]. These solitons were soon successfully demonstrated in a series of experiments [28–30, 32]. Figure 1 shows a typical example of a 2D screening soliton propagating through a biased photorefractive strontium barium niobate (SBN) crystal [28, 29]. In addition to steady-state screening solitons, many other families of photorefractive solitons (e.g. photovoltaic solitons) were subsequently reported [33, 34], all emerging from the rich behavior of the photorefractive effects [35].

The discovery of photorefractive solitons was important for many reasons. For example, the power necessary to generate these solitons can be as low as a few microwatts (insensitive to the absolute intensity), thus enabling many experiments to be carried out with weak CW laser beams. Another unique feature of photorefractive solitons is that a weak soliton beam can induce a waveguide that can be used in turn to guide other more intense beams at wavelengths that are photorefractively less sensitive [36–38]. This makes them attractive for waveguiding and steering applications. In particular, it has been demonstrated in a series of experiments that photorefractive soliton-induced waveguides can be used for device applications such as directional couplers and high-efficiency frequency converters [39, 40].

2.2. Quadratic solitons

Another general class of spatial solitons that has been experimentally demonstrated was that of quadratic solitons. Quadratic solitons were predicted in the 1970s in quadratic nonlinear media [41], and were experimentally demonstrated in 1995 [42]. In this case, the beam-trapping mechanism arises because of the energy exchange between the fundamental and second harmonic, as described by the usual coupled mode

equations for second harmonic generation (SHG). The initial experiment used type II phase-matching in KTP, i.e. it involved two orthogonally polarized fundamental input beams. The crystal geometry was such that the extra-ordinary fundamental wave and the second harmonic ‘walked away’ from the ordinary fundamental beam. That is, the group velocities of the interacting beams were not collinear. It was demonstrated, however, that once the soliton was formed on and near phase-matching conditions, the fundamental and generated second-harmonic fields were mutually trapped as a result of the strong nonlinear coupling which counteracted both beam diffraction and beam walkoff. Furthermore, even though a steady-state quadratic soliton consists of in-phase fundamental and harmonic fields, they can also be generated during the SHG process using a fundamental beam input only. The importance of this particular type of interaction is that it demonstrated experimentally that nonlinear wave mixing itself can lead to soliton formation. In fact, it was shown that any parametric process involving the product of two finite beams could lead to mutual self-focusing, and presumably spatial solitons.

About the same time, (1+1)D quadratic solitons were demonstrated in LiNbO₃ waveguides [43, 44]. What distinguishes this experiment from previous work is the amount of second harmonic generated. In this case the level of second harmonic was small, since the geometry chosen was far from the phase-matching condition. This limit is often called the cascading or Kerr limit in which only a small amount of second harmonic is needed to impart a nonlinear phase shift to the fundamental beam proportional to its intensity. This leads to beam self-focusing, and quadratic soliton formation can take place similar to that expected in the Kerr case. This 1D experiment also indicated that the effective nonlinearity also depends on the phase mismatch, which can thus be tunable both in sign and magnitude. In addition, the geometry can be flexible because stringent phase-matching conditions need not be imposed. It is these concepts that ultimately led to quadratic solitons in which temporal spreading was also arrested, at least in one dimension. For a detailed review about quadratic solitons, please refer to [45].

2.3. Spatial solitons in nonlocal and other nonlinear media

We briefly mention here that, in addition to the basic Kerr (cubic), photorefractive and quadratic nonlinearities already discussed, there are many other families of such nonlinearities via which optical spatial solitons can form. Some typical examples of such nonlinear mechanisms include resonant nonlinear effects in atomic vapors [46], nonlinear upconverted photobleaching in dye-doped polymers [47, 48], quadratic electro-optic effects in paraelectric nonlinear crystals [49], orientational enhanced photorefractive in organic nonlinear materials [50, 51], orientational [52] and thermal nonlinear effects [53, 54] in liquid crystals and pyroelectric effects [55].

Among these, an important class of nonlinearities is that which is associated with nonlocal processes, which may occur, for example, in liquid crystals and thermal nonlinear media. Nonlocality in nonlinearity has profound effects on the dynamics of optical solitons, since it can lead to

a counterintuitive behavior, when compared with traditional Kerr-type solitons [56–64]. For instance, nonlocal solitons can overcome repulsion between two out-of-phase bright solitons or two in-phase dark solitons. Nonlocal solitons can also form bound states, as observed in both 1D and 2D settings. In addition, it has been demonstrated that nonlocality can lead to new families of waves and soliton interaction dynamics that would have been otherwise impossible in local isotropic nonlinear media, including solitons in nonlinear media with an infinite range of nonlocality, long-range interactions between solitons and nonlocal surface solitons. Some of these newly studied soliton phenomena will be mentioned in section 5.

3. Different families of spatial solitons

Regardless of the origin of nonlinearity, one can classify spatial solitons into broader categories based on their inherent characteristics. Such broad categories may include, for example, that of bright and dark solitons, single- and multi-component solitons, coherent and incoherent solitons, spatial and spatiotemporal solitons, traveling wave and cavity solitons, as well as continuous and discrete solitons. In this section, we provide a brief review of a few such different families of spatial solitons. The general area of discrete solitons will be discussed separately in the next section.

3.1. Dark solitons and optical vortex solitons

Dark spatial solitons represent self-trapped optical beams involving a dark ‘notch’ (in the 1D case) or a ‘hole’ (in the 2D case) in an otherwise bright background. A 1D dark soliton is typically characterized by a π phase shift across the dark line where the field is zero, while a 2D dark soliton is a vortex soliton manifested by an azimuthal $2m\pi$ phase shift around the dark hole. As was theoretically shown in 1973, 1D dark solitons should also exist in materials with self-defocusing nonlinearities, in stark contrast to bright solitons [65]. 2D dark vortex solitons, studied even earlier within the context of super-fluidity [66], were also predicted to occur in nonlinear optics [67].

The first experiments on dark solitons were performed around 1990 and 1991 [68, 69]. These experiments employed a variety of media, including those with thermal and semiconductor nonlinearities, all of which were of the saturable type. In the early work of Schwartzlander and co-workers, spatial dark-soliton stripes and grids were experimentally observed in the transverse plane of a laser beam propagating in a self-defocusing nonlinear medium. Shortly after this work, stable propagation of a 2D dark vortex soliton was also observed in self-defocusing media with thermal nonlinearities [70]. The vortex was experimentally created using a quasi-helical phase mask, and it propagated without any change in shape under self-defocusing conditions, except for a phase rotation around its center of symmetry as imposed by its azimuthal phase. In addition to dark-soliton stripes and vortices, gray spatial solitons (with a phase change across the notch of less than π) that have a finite transverse velocity were also demonstrated [71, 72].

After the first prediction and experimental observation of bright photorefractive solitons [23, 24], steady-state *dark* photorefractive screening [25, 26] and photovoltaic [33] solitons were also predicted. The first attempt to observe 1D dark photorefractive solitons and 2D vortex solitons was made by Duree *et al* in a bulk photorefractive crystal [73]. These dark solitons were of the nonlocal type, and were observed in quasi-steady state. Shortly after, steady-state 1D dark screening solitons [30, 74, 75] and dark photovoltaic solitons [76] were also observed. While 2D dark vortex solitons were anticipated to exist and were indeed observed in isotropic self-defocusing media [70, 77], it was not immediately obvious that the anisotropic photorefractive nonlinearity would also support isotropic structures such as dark soliton grids and circular vortex solitons. In the experiments of [78, 79], steady-state photorefractive vortex solitons were successfully demonstrated by employing the screening nonlinearity in a biased SBN crystal as well as the photovoltaic nonlinearity in an unbiased LiNbO₃ crystal, despite the inherent anisotropy of the photorefractive nonlinearity. Interestingly, pairs of optical vortex solitons were also generated from the instability and breakup of a dark soliton stripe in both isotropic [80] and anisotropic [81] saturable self-defocusing nonlinear media.

Dark and vortex solitons have a unique place in nonlinear optics [82, 83]. The fact that plane waves are unstable in self-focusing media and can therefore disintegrate into bright solitons is perhaps not surprising. However, plane waves are stable in self-defocusing media. Therefore, an extra ‘ingredient’ (phase jump or phase singularity) is needed to form 1D dark solitons or 2D vortex solitons, as shown in all experimental demonstrations in homogeneous nonlinear media. This phase feature associated with dark solitons has played an important role in soliton phenomena and in particular in vortex soliton dynamics in discrete nonlinear systems.

3.2. Vector (multi-component) spatial solitons

Vector solitons refer to solitons involving multiple components that are not only coupled together but that are absolutely necessary in order to maintain their shapes during propagation. From a theoretical perspective, the existence of a vector soliton was first predicted in 1974 by Manakov [84], who studied two degenerate solitons that are polarized along two orthogonal axes and coupled through equal self-phase and cross-phase modulation effects. An experiment on two-component vector solitons was carried out by Shalaby and Barthelemy in 1992, who demonstrated that a bright–dark spatial soliton pair can propagate in a nonlinear material such as CS₂ [85]. This composite structure involved two different soliton components coupled through cross-phase modulation between the two different wavelengths. Although vector solitons were also suggested in the temporal domain (i.e. in fibers) [86–88] and later on for spatial solitons from a different perspective [89], it was not until 1996 that the experimental field of multi-component vector solitons actually blossomed.

The first experimental demonstration of vector solitons in the form proposed by Manakov was performed in AlGaAs waveguides in 1996 [90]. When the electric field

vectors are polarized parallel to AlGaAs 110 and 001 crystalline axes, it so happens that the self- and cross-phase modulation terms are approximately equal, thus satisfying the requirement for Manakov solitons. In that experiment, the two polarizations were passed through two separate optical systems with different group velocity dispersions in order to eliminate the temporal coherence between them. Over the years, two additional methods (other than those relying on orthogonal polarizations) were suggested to realize multi-component solitons. The first one assumes that the two soliton components are widely separated in the frequency scale [91], whereas the second one considers components which are mutually incoherent with respect to each other [92]. The latter method proves particularly useful in terms of implementing *N*-component Manakov-type solitons. This is possible in materials with non-instantaneous nonlinearities (as in photorefractive crystals) even when all components share the same wavelength and polarization. This method was employed with photorefractive solitons, first to demonstrate two-component bright–bright, dark–dark and dark–bright soliton pairs [93, 94], and later on to demonstrate 1D multi-mode/multi-hump solitons [95] and 2D multimode solitons consisting of a bell-shaped component and a dipole mode [96, 97].

The general ideas behind multi-component vector solitons proved invaluable for later developments and in particular to the area of incoherent solitons discussed in the next section.

3.3. Incoherent solitons

Incoherent solitons are self-trapped partially coherent wave entities propagating in nonlinear media. Before 1996, incoherent solitons were thought to be impossible because solitons were considered to be exclusively coherent entities. In 1996, self-trapping of partially spatially incoherent light was first observed in experiment [98] by Segev’s group, followed by the observation of self-trapping of both temporally and spatially incoherent white light [99]. Yet in another experiment shortly after, self-trapping of 1D dark incoherent beams along with 2D dark incoherent beams (with 2D ‘voids’ nested in spatially incoherent beams) was also demonstrated [100]. These experimental demonstrations completely changed the commonly held belief that solitons could only be formed from coherent waves [101, 102]. Figure 3 shows the experimental results concerning white-light bright solitons and incoherent dark solitons.

From a waveguide viewpoint, an incoherent soliton forms when its time-averaged intensity induces a multimode waveguide and then traps itself in it by populating the corresponding guided modes in a self-consistent fashion. These random-phase and weakly correlated self-trapped entities exhibit a host of unique properties that have no analog in the coherent regime. Moreover, their existence is related to many other areas of physics, in which nonlinearities, stochastic behavior and statistical averaging are involved. To describe the formation of incoherent solitons in non-instantaneous nonlinear materials, several theoretical approaches were developed, including the coherent density

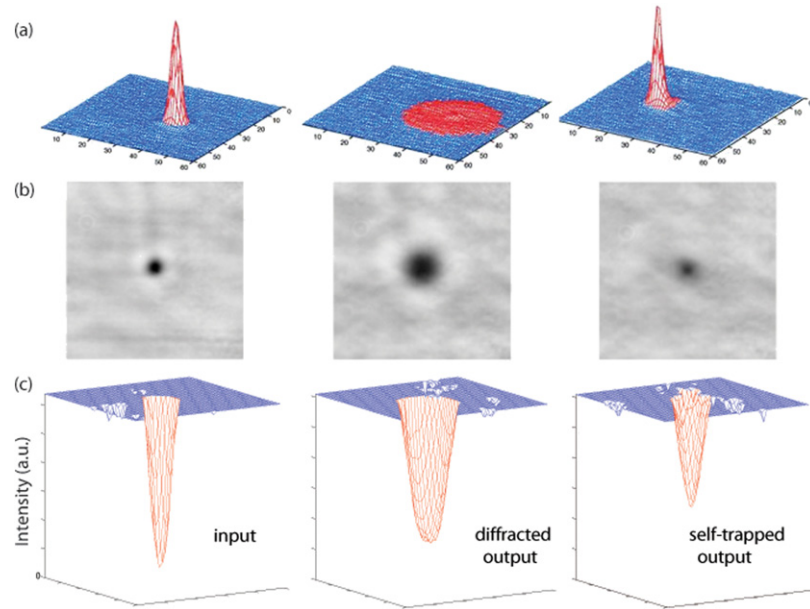


Figure 3. Experimental demonstration of self-trapping of (a) a 2D incoherent white-light beam and (b), (c) of a partially incoherent dark (vortex) beam. The top panels (a) show measured 3D intensity patterns of a white-light beam at input, linear output and nonlinear output positions, and the middle and bottom panels show 2D transverse patterns and 3D intensity plots of an incoherent vortex beam [99, 100]. Reproduced with permission from [100]. Copyright 1998 American Association for the Advancement of Science.

theory, the modal theory and the mutual coherence theory [103–107]. Meanwhile, the theory describing the self-trapping of incoherent dark beams was also established, revealing that incoherent dark solitons have an inherent grayness because they consist of both bound and radiation modes [108, 109].

These experimental and theoretical studies clearly showed that bright and dark solitons can exist with both spatial and temporal coherence, and laid the basis for subsequent research on incoherent solitons and incoherent nonlinear wave dynamics in general. A host of new phenomena mediated by incoherent waves were subsequently uncovered. These include, for example, dark soliton splitting and ‘phase memory’ effects [110], anti-dark incoherent soliton states [111], and incoherent modulational instability [112–114]. Following this, a number of experiments were carried out demonstrating the effects of partial coherence on various soliton-related nonlinear wave phenomena. In particular, spontaneous clustering of solitons in partially coherent wavefronts [115] was demonstrated. The clustering phenomenon is an outcome of the interplay between random noise, weak correlation and high nonlinearity, which has no counterpart with solitons in coherent systems. Another example is the formation of spatial soliton pixels from partially coherent light [116] as illustrated in figure 4, whereas closely spaced spatial solitons are difficult to realize with coherent light. The rapid progress in the new area of incoherent solitons also set up the basis for the later advancements in incoherent (random-phase) solitons in discrete nonlinear systems.

3.4. Spatiotemporal solitons

Spatiotemporal solitons are simultaneously confined wavepackets of radiation in both space and time, often termed as ‘optical bullets’. While both temporal and spatial solitons have been

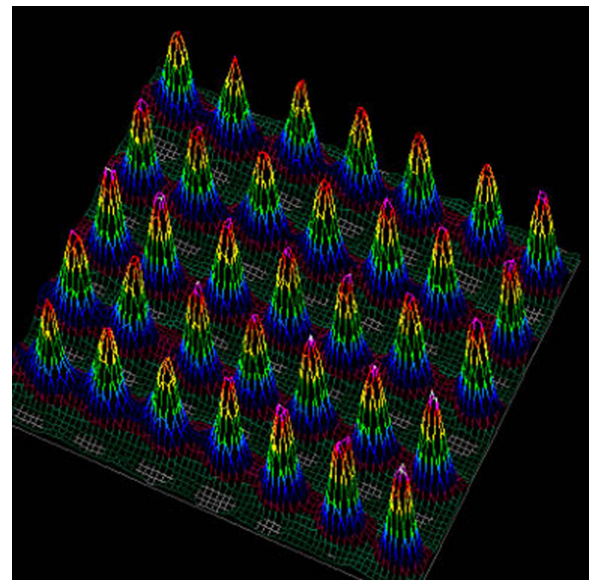


Figure 4. Experimental demonstration of spatial soliton arrays of partially incoherent light. Reproduced with permission from [116]. Copyright 2002 The Optical Society.

well known since the early 1970s, the possibility of optical bullets was not suggested until 1990 [117]. Since then, there have been many attempts in generating such optical bullets, but experimental demonstration was hampered due to the difficulty in finding the right material so that, for a given optical pulse width, the dispersion length in time matches the diffraction length in space as well as the nonlinear length. In fact, in ordinary materials such as glass, a precise balance of the nonlinear and linear effects is hard to maintain, as a slight variation in pulse or material parameters can easily destroy such a balance.

The first experimental work reporting a ‘quasi-bullet’ used clever schemes to control the GVD along one spatial

axis [118]. In that experiment, the spatiotemporal solitons observed were 2D in nature: they were confined in time and one transverse spatial dimension, but underwent linear diffraction in the second dimension. The experiment used quadratic soliton interactions in the cascaded limit in bulk LiIO₃ with highly elliptically shaped beams. The pulse width of 110 fs was used, with a grating engineered GVD, to match the dispersion length to the diffraction length in one transverse dimension. Ultimately, however, such solitons are unstable at high intensity levels due to the lack of confinement along the second spatial dimension, and thus a collapse into a number of filaments occurs. This first experiment further stimulated the experimental search for light bullets [119]. X waves were also proposed as a means to resist the effects of diffraction and dispersion but by their very nature are not localized (they have an infinite norm) [120]. Nevertheless, recent advances in spatiotemporal solitons as well as intelligent beam engineering have opened the road to ‘true’ 3D optical bullets as we shall mention in section 5.

3.5. Dissipative spatial solitons and cavity solitons

Dissipative solitons are self-trapped structures occurring in non-conservative systems, which require a nonlinear balance between loss and gain in addition to that between nonlinearity and dispersion/diffraction [121]. Dissipative solitons may arise as bright spots in a 2D transverse plane orthogonal to the beam propagation direction, often in an optical pattern-forming system with or without feedback. They have been studied in a number of experimental settings [122–127].

A typical example of such an entity is the so-called cavity soliton, which appears as a single-peak localized structure trapped between reflecting surfaces in contrast to other solitons that are traveling waves. In semiconductor devices, cavity solitons have been predicted to exist as spatial soliton pixels [122] and have been experimentally observed [127] in broad-area vertical cavity surface emitting lasers (VCSELs) driven by injection of a coherent and homogeneous field as a holding beam. This introduces new properties and significantly widens the class of materials in which soliton phenomena may be explored. Although there have been a number of experiments reported on patterns in resonators, evidence of localization of structures as isolated solitons was reported with a passive semiconductor microresonator, one of the simplest nonlinear optical feedback systems [125]. Requirements for the formation of solitons or localized structures were investigated in the spectral range where the nonlinearity of the semiconductor material is dispersive and defocusing. Another experiment reported stable, controllable spatial solitons in Na vapor with a single feedback mirror [126]. However, a clear experimental demonstration of cavity solitons was carried out with VCSELs that were electrically pumped above transparency but slightly below the lasing threshold [127], in which the generated cavity solitons could be written, erased and manipulated as objects independent of each other and of the boundary.

The progress on cavity solitons, especially in systems that do not support traveling-wave states (e.g. dissipative systems), is expected to bring up new possibilities with spatial

solitons. Some recently studied examples include cavity soliton lasers [128, 129], Bloch cavity solitons [130] and cavity polariton solitons [131].

4. Spatial solitons in discrete systems

Linear and nonlinear discrete or periodic systems are abundant in nature. In optics, a typical example of such an arrangement is that of a closely spaced waveguide array, in which the collective behavior of wave propagation exhibits many intriguing and unexpected phenomena that have no counterpart in homogeneous media. In such discrete systems, another fascinating class of self-trapped states, spatial discrete solitons (or lattice solitons), have been uncovered and have become the mainstream of soliton research in the past decade. These solitons arise from the interplay between discreteness and nonlinearity. Since the first prediction [132] and experimental demonstration [133] of 1D discrete solitons, this field has grown rapidly. For a detailed review, please refer to [134–136]. Below we just provide a brief overview of some key experimental works on discrete solitons carried out in 1D AlGaAs nonlinear waveguide arrays and 2D optically induced photonic lattices.

4.1. Discrete solitons in 1D waveguide arrays

A discrete soliton can form as a result of a balance between discrete diffraction and nonlinear self-focusing effects in periodic waveguide arrays, as first predicted by Christodoulides and Joseph in 1988 [132]. Unlike other families of spatial solitons, which are known to exist in homogeneous media, discrete solitons result from the collective behavior of the array as a whole. In reality they represent nonlinearity induced defect modes in a photonic crystal or a photonic lattice [134–136]. In the pioneering experiment of Eisenberg *et al* a 1D discrete soliton was observed in an AlGaAs nonlinear waveguide array [133], in which the light became trapped to only a few waveguides through the Kerr self-focusing nonlinearity, as opposed to linearly spreading laterally due to evanescent wave coupling (figure 5). This first observed discrete soliton existed in the so-called semi-infinite gap. This is because of the nonlinear induced refractive index change, so the propagation constant of the soliton itself was locally elevated to the semi-infinite gap under the self-focusing nonlinearity. In subsequent experiments, Morandotti *et al* studied discrete soliton transport as well as self-focusing and defocusing dynamics in such 1D waveguide arrays [137, 138] based on the normal and anomalous diffraction properties of the system [139, 140]. Much of the earlier theoretical and experimental work on 1D waveguide arrays has been reviewed in [141, 142]. The bandgap structures and peculiar refraction and diffraction properties of discrete optical systems provided many new possibilities for spatial solitons that could not otherwise occur in continuous optical systems [136].

An example unique to solitons in discrete systems is that of a *gap soliton*. In optics, gap solitons are traditionally considered as temporal phenomena in 1D periodic media such

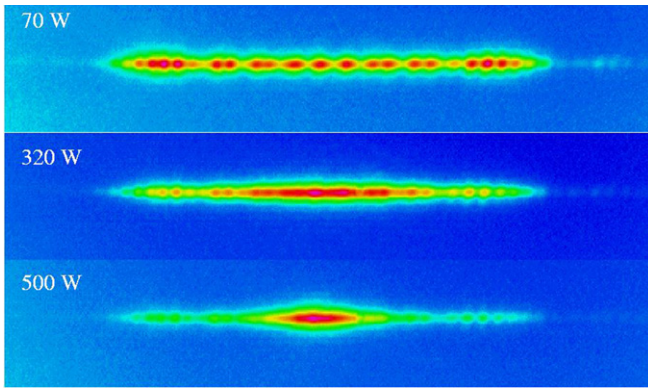


Figure 5. Experimental demonstration of a 1D discrete optical soliton in AlGaAs semiconductor waveguide arrays. Images are taken from the output facet of a 4 mm long sample for different powers. At low powers, the beam displays linear diffraction, but at high powers, a discrete soliton is formed. Reproduced with permission from [133]. Copyright 1998 American Physical Society.

as fiber gratings [143–145]. The existence of spatial gap solitons in waveguide arrays was suggested by Kivshar in 1993 [146]. In contrast to discrete solitons existing in the semi-infinite gap, these latter spatial gap solitons require a balance between anomalous diffraction and self-defocusing nonlinear effects. In this case, the nonlinear index change makes the soliton propagation constant lie somewhere between the first and the second optical Bloch bands (within the first gap), near the edge of the first Brillouin zone (BZ) of a waveguide array. Such gap solitons have a ‘staggered’ phase structure, as first observed in 1D photonic lattices optically induced in a photorefractive nonlinear crystal by Segev’s group [147]. In subsequent experiments, 1D Floquet–Bloch gap solitons [148], reminiscent of those in Bragg gratings [143], and the generation and steering of spatial gap solitons originating from higher Bloch bands [149, 150] were also demonstrated. Another process unique to solitons in discrete systems is that associated with discrete diffraction managed spatial solitons [151].

In addition to fundamental discrete solitons, other families of spatial solitons and related nonlinear phenomena studied in the bulk have also been investigated in discrete systems, including dark discrete solitons [138, 152], discrete modulation instability [153] and discrete vector solitons [154]. As we will see, the first experimental demonstration of 2D discrete solitons [155] opened up new opportunities in exploring the rich physics of nonlinear periodic systems.

4.2. Discrete solitons in 2D photonic lattices

Although 1D waveguide arrays can serve as a test bench for studying many fascinating effects, it became increasingly clear that the opportunities offered by 1D arrays were rather limited, while rich soliton phenomena could arise in a high-dimensional environment. For instance, it has been suggested that, in 2D discrete waveguide networks, discrete solitons can be effectively routed or totally blocked using soliton collisions [156]. Yet, it has always been a challenge to fabricate 2D waveguide arrays with appropriate nonlinearity in bulk media.

The first experimental observation of a 2D optical discrete soliton was made in a biased photorefractive crystal by Segev’s and Christodoulides’ groups [155]. Much of their success relied on a key idea, suggested by Efremidis *et al* from these same groups in 2002 [157], of optically inducing defect-free 2D nonlinear photonic lattices in biased photorefractive crystals such as SBN. The suggested optical induction method made use of the large electro-optic anisotropy of these nonlinear crystals and the associated photorefractive screening nonlinearity used earlier for the generation of photorefractive solitons [25–30, 32]. In such arrangements, the nonlinear index change induced and experienced by an optical beam depends on its polarization as well as on its intensity. Under appreciable bias conditions, this index change for an extra-ordinarily polarized beam can be 10 or more times larger than that for an ordinarily polarized beam, due to the large difference between the electro-optic coefficients in the nonlinear photorefractive crystal. Thus, if the lattice-inducing beam is *o*-polarized while the soliton-forming beam is *e*-polarized, the lattice beam would induce only a weak index change and could be considered as undergoing linear propagation, while the soliton-forming beam can not only experience the periodic optically induced lattice but can also exhibit nonlinear behavior. Using this mechanism, Fleischer *et al* created a 2D invariant photonic lattice in an SBN:75 crystal by interfering multiple plane-wave beams and observed the 2D discrete spatial solitons as well as 2D gap solitons [155].

The optical induction method was subsequently used by many other teams. In particular, Kivshar’s group observed twisted soliton modes in 1D optically induced gratings [158], and Chen’s group demonstrated discrete soliton-induced dislocations in 2D partially coherent lattices [159]. The latter established 2D photonic lattices based on the amplitude modulation of a partially coherent beam rather than multiple-beam interference, with active control of the *Talbot* effect of the lattice-inducing beam. In such partially coherent lattices, a clear transition from 2D discrete diffraction to discrete solitons was observed and recorded as the nonlinearity was gradually increased. Figure 6 shows typical experimental results of 2D discrete solitons observed in 2D photonic lattices induced with partially coherent light. The method of optical induction based on partially coherent light provided an effective way for inducing nonlinear photonic lattices [116] and later on for inducing various reconfigurable lattices with structured defects and surfaces [160, 161].

Apart from fundamental discrete solitons in 2D lattices, discrete vortex solitons with topological phase singularities were also proposed [162, 163] and soon after demonstrated experimentally in the same setting of such optically induced photonic lattices [164, 165]. These singly charged vortexing solitons have a nontrivial $\pi/2$ step-phase structure, as confirmed experimentally by interference phase measurements (figure 7). Shortly after, higher band vortex gap solitons were also identified and observed in 2D induced lattices [166, 167], as well as higher charge [168, 169] and necklace-type [170] self-trapped vortex structures. In fact, using such a lattice reconfiguration under both self-focusing and self-defocusing nonlinearities [155, 171–173], a host of discrete

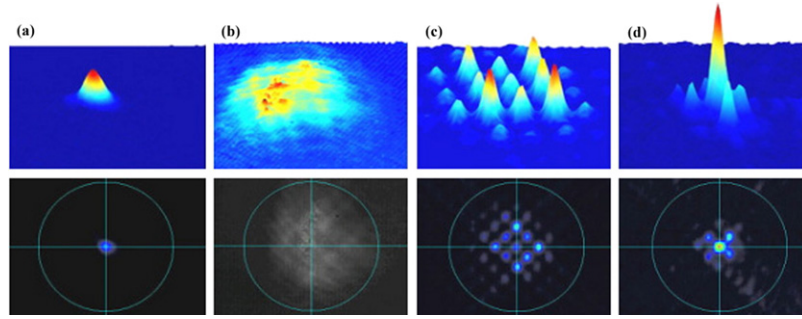


Figure 6. Experimental demonstration of a 2D discrete spatial soliton in an optically induced photonic lattice. (a) Input, (b) diffraction output without the lattice, (c) discrete diffraction at low nonlinearity and (d) discrete soliton formation at high nonlinearity. Top panels: 3D intensity plots; bottom panels: corresponding 2D transverse intensity patterns. Reproduced with permission from [159]. Copyright 2004 American Physical Society.

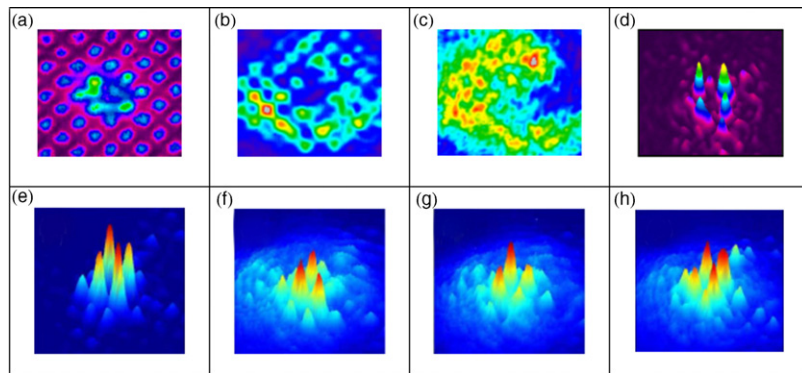


Figure 7. Experimental demonstration of discrete vortex solitons. (a)–(d) On-site vortex beam in which the singularity is centered on a waveguide site, where (a) shows input, (b) linear diffraction output, (c) an interferogram formed by interfering the output beam (b) with a plane wave showing the characteristic $0 \rightarrow 2\pi$ phase structure of a vortex, and (d) nonlinear output of vortex-ring lattice soliton. Reproduced with permission from [165]. Copyright 2004 American Physical Society. (e)–(h) Off-site vortex lattice soliton in which the singularity is located between sites, where (e) shows the soliton output, and (f)–(h) are interferograms formed when the phase of the plane wave is successively changed in steps of $\pi/2$. Reproduced with permission from [164]. Copyright 2004 American Physical Society.

soliton phenomena was demonstrated in various experimental settings. These included, for example, 2D dipole and vector lattice solitons [172, 173], discrete soliton trains [174, 175], incoherent (random phase) lattice solitons [176, 177], rotary solitons in Bessel-type ring lattices [178, 179], solitons in quasi-crystal lattices and honeycomb lattices [180, 181], ‘reduced-symmetry’ solitons [182], in-gap and in-band (‘embedded’) soliton trains [183, 184] and discrete surface solitons, which we shall discuss separately in the section that follows. Other research conducted in 2D induced lattices includes, for example, the development of the Brillouin-zone spectroscopy method [185], spatial four-wave mixing and super-continuum generation [186, 187], bandgap guidance in lattices with low-index cores and structured defects [188–190], Bloch oscillations and Zener tunneling [191], and the seminal experiment on transport and Anderson localization in disordered photonic lattices [192].

Another major step in 2D photonic structures is the recent development of high-precision femtosecond-laser writing of waveguide arrays in silica, which also enabled the observation of discrete solitons, as reported by Szameit *et al* [193, 194]. This development is important not only because it provided a promising alternative for fabricating 2D arrays, but also because it led to a powerful platform for ‘fabricating’ photonic lattices of various configurations—virtually any periodic or

aperiodic network of waveguide arrays could be realized using this method.

4.3. Discrete surface solitons

Since Tamm and Shockley introduced electronic surface waves in the 1930s, surface wave phenomena have been of continuing interest in various areas of physics. In optics, nonlinear stationary surface states were actively considered in the early 1980s. However, progress in directly observing such nonlinear surface states was hampered by issues such as high power requirement and instabilities.

In the context of discrete systems, it is of course natural for one to ask as to whether optical self-trapped waves can exist at the surface of a photonic lattice or at the interface between two different photonic structures. Along these lines self-trapped surface waves (optical surface solitons) were soon proposed and experimentally demonstrated. Such states can be loosely interpreted as nonlinear defect modes with propagation constants (or eigenvalues) located within the forbidden optical bandgaps of a periodic structure. 1D in-phase surface solitons were first predicted to exist at the edge of nonlinear self-focusing waveguide lattices by Stegeman’s and Christodoulides’ groups [195], and were demonstrated experimentally shortly after by the same

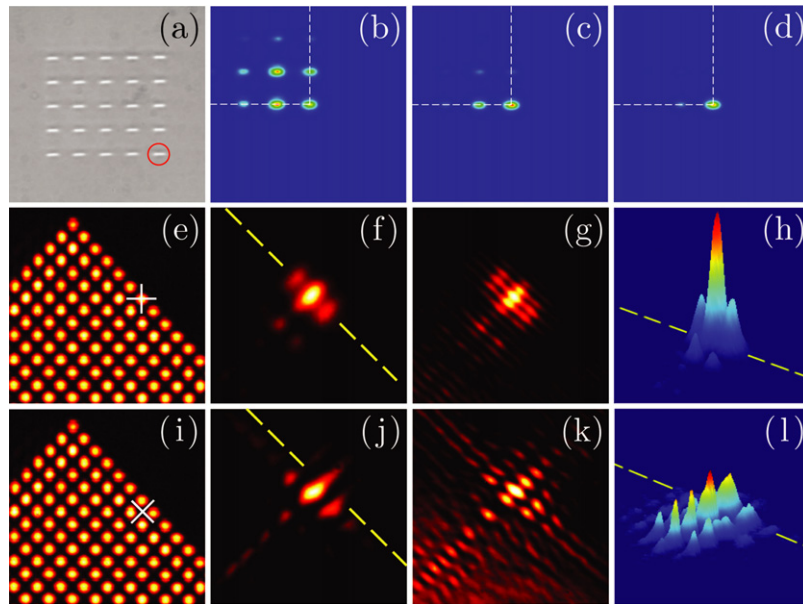


Figure 8. Experimental demonstration of 2D surface solitons. (a) Microscope image of a laser-written array with an excited waveguide marked by a circle. (b)–(d) Output intensity distributions for progressively increasing input power levels. Observation of surface soliton (middle row) and surface gap soliton (bottom row) in an optically induced photorefractive lattice. (e), (i) Lattice patterns with the waveguide excited by the probe beam marked by a cross. (f), (j) Surface soliton intensity patterns. (g), (k) Interference pattern between the soliton beam and a tilted plane wave. (h) 3D intensity plots of an in-phase surface soliton and (l) the corresponding pattern when its intensity is reduced significantly under the same bias condition. In all plots, dashed lines mark the interface. (b)–(d) Adapted with permission from [207]. Copyright 2007 American Physical Society. (e)–(l) Reproduced with permission from [206]. Copyright 2007 American Physical Society.

group in AlGaAs arrays with a dominant Kerr nonlinear effect [196]. Meanwhile, 1D surface gap or ‘staggered’ solitons at the interface between a uniform medium and a self-defocusing waveguide array were also proposed [195, 197], and demonstrated in both quadratic [198] and saturable photorefractive nonlinear media [199, 200]. Unlike their in-phase counterparts, these latter entities have their propagation eigenvalues located in the first photonic bandgap (between the first and second Bloch bands) at the edge of the BZ. These studies extend the correspondence between optical surface waves and localized surface Tamm states into the nonlinear regime, since surface gap solitons can be viewed as the optical nonlinear analogs of Tamm states [201].

In the 2D domain [202–205], a direct experimental observation of 2D surface solitons remained a challenge due to experimental difficulties in fabricating 2D nonlinear lattices with sharp surfaces or interfaces. In 2007, two independent experimental demonstrations of 2D surface lattice solitons were reported using different materials and settings: one was accomplished by Wang *et al* in optically induced lattices in a photorefractive crystal [206], while the other was carried out by Szameit *et al* in femtosecond-laser-written waveguide arrays in bulk fused silica [207]. Figure 8 shows typical experimental results of 2D discrete surface solitons obtained from these two independent studies. In the fs-laser-written waveguide experiment, focused laser pulses were sent into bulk fused silica, which created a localized permanent increase in the refractive index of the material [194]. Consequently, when moving the sample transversely with respect to the beam, a longitudinal extended index modification (a waveguide) was written. A microscope image of the facet of such a laser-written

5×5 waveguide array is shown in figure 8(a). While for low input peak powers a clear spreading of the light into the array was observed, for a high input peak power almost all of the light was localized in the excited waveguide [207]. In the photorefractive induction experiment, the lattice pattern was generated by a periodic modulation of a partially incoherent optical beam with an amplitude mask, which was then sent into an SBN crystal to induce a square lattice featuring sharp edges or corners. With an appropriate high bias field, the spreading of a probe beam was suppressed at the lattice interface to form a discrete surface soliton and a surface gap soliton, while the beam at reduced intensity displayed significant diffraction under the same lattice conditions [206].

Over the last few years, considerable work has been devoted to discrete surface solitons [136, 201]. Much of the work theoretically investigated surface solitons of different types such as vector, vortex, incoherent, polychromatic and spatio-temporal surface solitons [203, 208–212]. Given that the geometries of the lattice surfaces and interfaces can vary considerably, discrete surface solitons were observed in a number of different settings including the interface between two dissimilar periodic media and superlattice surfaces [213–216]. In addition to nonlinear Tamm-like surface states, linear optical Shockley-like surface states were first introduced and observed in optically induced photonic superlattices [217, 218], and in subsequent experiments, transitions between Shockley-like and nonlinear Tamm-like surface states were also demonstrated. Furthermore, surface states that do not belong to the same family of Tamm or Shockley states have also been demonstrated as a new type of defect-free surface states in fs-laser-written curved waveguide arrays [219, 220].

4.4. Discrete solitons in other material systems

In addition to the aforementioned intensity-dependent discrete media (AlGaAs semiconductors, optically induced photonic lattices and fs-laser-written waveguides), spatial discrete solitons were also studied and experimentally observed in many other material systems. In particular, Lederer's group theoretically investigated the formation of quadratic discrete solitons based on the cascaded nonlinearity [221, 222], and subsequently, these solitons along with discrete modulation instability were observed by Stegeman's group in quadratic nonlinear waveguide arrays [223, 224]. Discrete solitons were also observed in nematic liquid crystal cells by Assanto's group [225, 226], and in defocusing photovoltaic LiNbO₃ waveguide arrays by Kip's group [227]. Other families of spatial solitons previously studied in continuous systems such as cavity solitons and dissipative solitons were also suggested in discrete nonlinear arrangements [228, 229]. Discrete solitons and nonlinear localized modes were studied in photonic crystal waveguides [230] and nonlinear coupled microcavities embedded in photonic crystals [231]. Many other theoretical and experimental studies considered discrete solitons in various optical settings [134–136, 232, 233].

5. Recent developments in optical spatial solitons

Apart from a continuous growth of interest in spatial solitons in waveguide arrays and photonic lattices, other advances in optical spatial solitons are nowadays impacting fields like Bose–Einstein condensates, soft-condensed matter and plasmonics. Some recently discovered optical solitons or related novel phenomena include 3D light bullets, self-accelerating solitons propagating along curved trajectories, subwavelength plasmonic solitons in nonlinear metamaterials, soliton formation and self-induced transparency in nonlinear soft condensed matter, and spatial beam dynamics and optical solitons in nonlinear materials of parity–time (PT) symmetry.

5.1. New families of spatial solitons

As previously indicated the field has been constantly enriching itself with new ideas. Even during the period of writing this review, there have been many such developments in spatial soliton phenomena reported in the literature. Here, we present just a few samples of such recent activities.

Nonlocal incoherent solitons. Incoherent optical spatial solitons are self-trapped beams with a multimode structure that varies randomly in time, which occur in nonlinear homogeneous [98] and periodic [176, 177] materials with *non-instantaneous* nonlinearities. Apart from incoherent spatial solitons supported by nonlinearities with a slow response time (much longer than the characteristic fluctuation time of the beam, thus named 'non-instantaneous'), it has recently been demonstrated by Segev's group that incoherent solitons could also exist in effectively *instantaneous* nonlocal nonlinear media [234, 235]. These solitons exhibit fundamentally new features: they propagate along random trajectories and they can be created in various nonlinear optical media as well as in

other settings where the nonlinearity is nonlocal and very fast. This was somewhat surprising since it was believed that optical incoherent spatial solitons can exist only in noninstantaneous nonlinear media. The reason behind this was that only under such conditions could the nonlinear index change induced by a fluctuating beam become stationary in the propagation direction, which is necessary for guiding the multiple modes comprising the incoherent beam. Nonlocal nonlinearities can overcome this obstacle, as the trapping process is mediated by a highly nonlocal nonlinear response, which yields a spatial averaging instead of the traditional temporal averaging provided by the noninstantaneous response. Using this approach, nonlocal incoherent solitons were demonstrated not only in bulk media, but also at the interface between a linear medium and a highly nonlocal, effectively instantaneous nonlinear medium [236]. These incoherent nonlocal surface solitons exhibit features which are fundamentally different from their bulk counterparts or surface solitons created with local nonlinearities. Nonlocal incoherent solitons were also found to exist in spatially nonlocal nonlinear media with a logarithmic type of nonlinearity [237].

Polychromatic solitons. Polychromatic solitons are self-trapped optical beams involving multiple frequency components that are nonlinearly coupled together and propagate in the same direction. These solitons were previously studied theoretically as polychromatic partially spatially incoherent solitons in a noninstantaneous Kerr nonlinear medium, where a polychromatic incoherent soliton exists when its higher temporal frequency components are less spatially coherent than the lower frequency components [238]. Recently, there has been a great deal of interest in the study of multiple color beams, including the propagation of broad bandwidth and multi-color optical wavepackets in periodic photonic structures, under supercontinuum and white light generation conditions. In bulk nonlinear media, polychromatic or white-light solitons can only be supported by self-focusing nonlinearities. In photonic lattices, however, it was found that polychromatic solitons can exist under both self-focusing and -defocusing conditions. In particular, Neshev *et al* reported the experimental observation of discrete polychromatic solitons resulting from the localization of supercontinuum light through the nonlinear interaction of spectral components in extended periodic structures (figure 9) [239]. Polychromatic vortex solitons and discrete polychromatic surface solitons were demonstrated in subsequent experiments [240–242]. Meanwhile, in arrays of periodically curved waveguides, some fundamental processes occurring in solid-state physics such as dynamical localization have been introduced into optics and demonstrated in laser-written waveguide arrays [243–245]. In the nonlinear regime, polychromatic solitons have also been studied recently in curved waveguide arrays. When spectral components interact incoherently, the longitudinal modulation of the waveguide coupling can facilitate all optical switching of polychromatic light between two coupled waveguides, whereas in curved waveguide arrays nonlinearity leads to symmetry breaking and formation of polychromatic diffraction-managed solitons [246]. These results demonstrate new possibilities for tunable demultiplexing and spatial filtering of supercontinuum light.

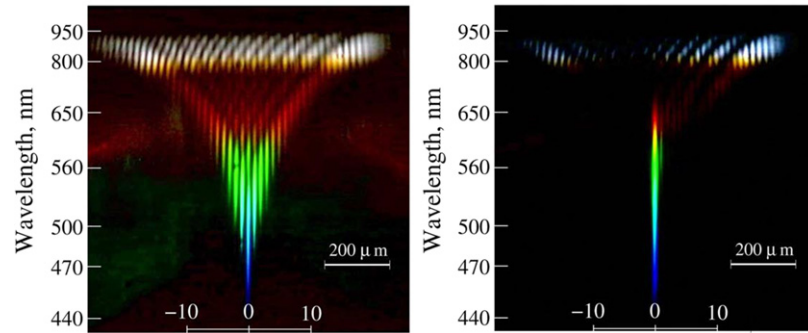


Figure 9. Experimental demonstration of spectrally resolved nonlinear localization of the supercontinuum in a waveguide array. At low powers, the supercontinuum beam diffracts linearly (left), but self-trapping is achieved at high powers (right). Reproduced with permission from [239]. Copyright 2007 American Physical Society.

‘Saddle’ solitons and hybrid nonlinearity. Periodic structures can exhibit several intriguing optical properties. In an optically induced 2D square lattice, for example, the high-symmetry X-point in the first Bloch band is akin to a ‘saddle’ point in the diffraction spectrum, where normal and anomalous diffractions co-exist along orthogonal directions. At this X-point, a quasi-1D soliton train can be excited provided that an appropriate type of nonlinearity is used to balance beam diffraction in one particular direction, whereas in the orthogonal direction it is an extended plane wave. The propagation constant of such a 1D soliton train could reside within the first Bloch band, thus termed ‘in-band’ or ‘embedded’ solitons [184]. However, to simultaneously balance normal and anomalous diffractions in different directions, one needs orientation-dependent hybrid nonlinearity. Such nonlinearity was identified by Zhang *et al* in nonconventional biased photorefractive crystals [247–251], in which a co-existence of self-focusing and -defocusing nonlinearities at two orthogonal directions can lead to a controlled 1D soliton transition from different band edges or subband edges [252]. This new setting enables the reconfiguration of photonic structures and BZs for band-gap engineering and light manipulation, including the control of Bragg reflection and the interplay between normal and anomalous diffraction/refraction under the same excitation conditions. Furthermore, with such hybrid nonlinearity, a new type of spatial gap solitons, namely, ‘saddle’ solitons, can be established. In fact, Hu *et al* observed such ‘saddle’ solitons by balancing the saddle-shaped bi-diffraction with hybrid focusing/defocusing in an optically induced 2D ionic-type lattice [253]. These ‘saddle’ solitons have a propagation constant residing in the Bragg reflection gap, but they differ from all previously observed solitons supported by a single focusing or defocusing nonlinearity.

Self-trapping of self-accelerating beams. Recently, a new class of nondiffracting beams, namely self-accelerating *Airy beams*, has been suggested by Christodoulides’ group [254–256]. In contradistinction with Bessel beams, Airy beams do not rely on simple conical superposition of plane waves. More importantly, they can self-accelerate during propagation in addition to being nondiffracting and self-healing. Over the past few years, considerable research has been devoted to Airy beams, ranging from their linear control to nonlinear self-trapping, covering fundamental aspects and demonstrations

for proposed applications. In practice, all non-diffracting Airy beams must be truncated, to keep their power finite. Such truncated beams eventually diffract and lose their structure after a long enough propagation distance. To overcome this problem several teams considered methods by which nonlinear physical mechanisms could be used to nonlinearly self-trap Airy beams, very much as in the case of optical spatial solitons.

In particular, Chen’s group studied the behavior of Airy beams as they move from a nonlinear medium to a linear medium. It was shown that an Airy beam, initially driven by a self-defocusing nonlinearity, experiences anomalous diffraction and can maintain its shape in subsequent propagation. Conversely, its intensity pattern and acceleration cannot persist when driven by a self-focusing nonlinearity [257]. Fleischer’s group reported the experimental observation of self-trapping of Airy beams in nonlinear photorefractive media with a diffusion nonlinearity [258]. In this case the self-trapped wavepacket self-bends during propagation at an acceleration rate that is independent of the thermal energy associated with the diffusive nonlinearity, and they represent a typical example of Airy solitary-like wave formation using two-wave mixing. Quite recently, Segev’s group studied self-accelerating self-trapped beams in nonlinear optical media [259], exhibiting self-focusing and self-defocusing Kerr and saturable nonlinearities, as well as a quadratic response. In Kerr and saturable nonlinear media such beams are stable under self-defocusing and weak self-focusing, whereas for strong self-focusing the beams off-shoot solitons while their main lobe continues to accelerate. These self-trapped Airy-like accelerating beams in nonlinear media propagate along parabolic trajectories, different from all optical spatial solitons previously observed in continuum or discrete nonlinear media. Yet in more recent developments, self-accelerating beams have been extended to nonparaxial regimes, where one could expect solitons to travel along a circular trajectory [260–263]. Under the action of nonlinearity, one could perhaps envision that ‘optical boomerang’ like beams might be realized originating from such nonparaxial self-accelerating beams.

5.2. Filamentation and light bullets

As mentioned before, there has been a considerable effort in generating wavepackets that are localized in all 3D space–time dimensions (propagating free of diffraction and dispersion

effects). Generating a light bullet typically requires a nonlinear mechanism that can simultaneously balance diffraction and dispersion during propagation, thus forming a *spatiotemporal soliton* as first suggested by Silberberg [117]. Over the past two decades, researchers have explored a variety of alternative materials and techniques to generate light bullets. Quite recently, Chong *et al* explored a new approach based on a nondiffracting Bessel beam and a non-dispersing 1D Airy pulse, thereby demonstrating Airy–Bessel wavepackets as versatile linear light bullets [264]. For nonlinear light bullets, Burgess *et al* introduced a new form of stable spatiotemporal self-trapped optical packets stemming from the interplay of local and nonlocal nonlinearities [265], where self-trapped light beams in media with both electronic and molecular nonlinear responses can be established due to a decoupling of spatial and temporal effects for independent tuning. Panagiotopoulos *et al* proposed an approach to tailor the filamentation of intense femtosecond laser pulses with periodic lattices [266], where diffraction induced by the lattices provides a regularizing mechanism to the nonlinear self-action effects involved in filamentation, leading to a new propagation regime of intense lattice solitons.

It is also worth noting that, quite recently, Minardi and co-workers reported the first experimental observation of 3D light bullets excited by femtosecond pulses in a system featuring a quasi-instantaneous cubic nonlinearity and a periodic, transversally modulated refractive index [267]. A 40 mm long, closely spaced 2D hexagonal array of silica glass waveguides was fabricated, and 170 fs pulses at a wavelength of 1550 nm (for which the glass had anomalous dispersion) were launched into a single waveguide at the center of the array. As illustrated in figure 10 [268], an input pulse exhibited linear propagation in the array at low powers, spreading from the excited waveguide into several neighboring waveguides while the pulse profile broadened. However, at high powers, not only the pulse became localized in the original waveguide but also the output pulse duration became much shorter. These experimental results represent the first demonstration of light pulses simultaneously localized in space and time for distances longer than the characteristic lengths dictated by linear propagation.

5.3. Subwavelength spatial plasmon solitons

Spatial plasmon solitons were studied theoretically in Kerr slabs embedded between metal plates [269] and in discrete nonlinear metamaterials [270]. Solitons in nanoscale periodic structures consisting of metal and nonlinear dielectric slabs rely on a balance between tunneling of surface plasmon modes and nonlinear self-trapping. The evolution in such systems arises from the threefold interplay between periodicity, nonlinearity and surface plasmon polaritons, and is substantially different from that occurring in conventional nonlinear dielectric waveguide arrays. Later, Peleg *et al* [271] studied wave propagation in arrays of subwavelength waveguides with a sharp index contrast, and found a self-reviving soliton (*‘phoenix soliton’*) comprised of coupled forward- and backward-propagating light, originating solely

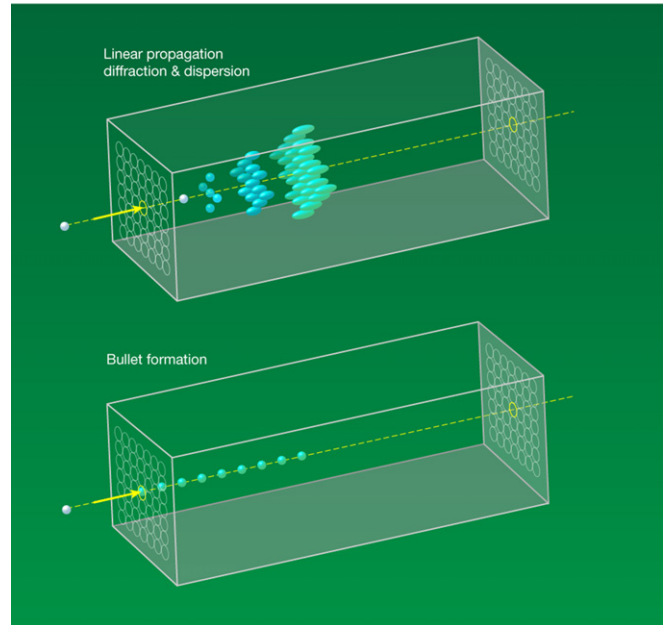


Figure 10. Illustration of 3D light bullets as observed in experiment with fs laser pulses propagating through a hexagonal array of glass waveguides. (top) A pulse undergoes spreading in both space and time during propagation in the waveguide array. (bottom) Nonlinear effects cause the light packet to remain stable and compact as it moves [267, 268]. Image courtesy of Alan Stonebraker. Reproduced with permission from [268]. Copyright 2010 American Physical Society.

from evanescent bands. In the linear regime, all Bloch waves comprising this wavepacket decay, whereas the nonlinearity can herd them into a propagating self-trapped beam. Recently, Ye *et al* [272] predicted theoretically that stable subwavelength plasmonic lattice solitons could be formed in arrays of metallic nanowires embedded in a nonlinear medium. The tight confinement of the guiding modes of the metallic nanowires, combined with the strong nonlinearity induced by the enhanced field at the metal surface, can provide the main physical mechanisms for balancing the wave diffraction and the formation of plasmonic lattice solitons. Davoyan *et al* [273] suggested a method for using tapered waveguides for compensating the losses of surface plasmon polaritons in order to enhance nonlinear effects on the nanoscale. They studied nonlinear plasmon self-focusing in tapered metal–dielectric–metal slot waveguides and demonstrated stable propagation of spatial plasmon solitons. The study of subwavelength solitons as nonlinear self-trapped plasmon modes is just beginning, and any success is expected to have important applications to subwavelength nonlinear nanophotonics.

5.4. Spatial solitons in soft condensed matter

Artificial nonlinear materials consisting of liquid suspensions of sub-micrometer dielectric particles have been of great interest since the early 1980s. Recently, a number of theoretical and experimental studies have considered nonlinear phenomena including modulation instability, spatial solitons and beam filamentation in nonlinear colloidal suspensions, and in soft condensed matter in general [274–279]. The

effects leading to self-channeling of light in fluid suspensions can be classified into two general classes: effects relying on light scattering, i.e. optical gradient forces, and thermal effects relying on (weak) absorption. Using thermal effects in liquids, the refractive index typically decreases with increasing temperature. Therefore, such systems can only support dark solitons. Recently, Segev's group predicted and observed a new type of self-trapped beam: a hot-particle soliton [280], formed in a nanoparticle suspension by virtue of thermophoresis. These experiments manifest a new kind of interplay between light and fluid. On the other hand, using scattering effects, most studies so far have considered colloidal suspensions of the polystyrene–water type. In these latter arrangements, the particles have a refractive index that is higher than that of the liquid, i.e. the suspension has a *positive* polarizability. Interestingly, it has recently been suggested that beam propagation in nanosuspensions with negative polarizabilities (when the index of the particles is lower than that of the liquid) can be stable and can exhibit unusual nonlinear optical properties such as self-induced transparency [281]. In fact, experimental demonstrations of self-trapping and enhanced transmission of an optical beam in nonlinear nanosuspensions exhibiting *negative* polarizabilities were reported very recently [282]. While self-focusing was observed in colloidal nanosuspensions with both positive and negative polarizabilities, self-induced transparency of an optical beam was realized only in colloidal systems with negative polarizabilities. These studies bring about many possibilities for studying nonlinear phenomena in nanosuspensions and soft condensed matter in general.

5.5. Spatial solitons in systems with PT symmetry

In 1998, it was proposed by Bender and Boettcher that a wide class of Hamiltonians, although non-Hermitian, can still exhibit real eigenvalue spectra provided that they obey the PT requirements or PT symmetry [283, 284]. The concept was introduced to optics ten years later by Christodoulides and co-workers, who investigated spatial beam dynamics in synthetic optical media with PT symmetries imposed by a balanced arrangement of gain or loss [285]. Experimental realizations of such PT systems have been reported recently, including observation of spontaneous PT symmetry breaking and power oscillations violating left–right symmetry in a PT optical coupled system involving a complex index potential [286], and observation of passive PT-symmetry breaking and phase transition that leads to a loss-induced optical transparency in specially designed pseudo-Hermitian guiding potentials [287]. These experimental results stimulated further interest in exploring new classes of PT-synthetic materials with intriguing and unexpected properties for fundamental research and novel applications. Nonlinear beam dynamics and spatial solitons in PT-symmetric potentials were also investigated theoretically. It was shown that a novel class of 1 and 2D nonlinear self-trapped modes can exist in optical PT-synthetic lattices [288]. In fact, spatial solitons in PT symmetry systems have attracted a great deal of attention recently, fueled by continued interest in spatial solitons, discrete phenomena

and PT-symmetric systems. For example, a number of theoretical studies have focused on the existence and stability of nonlinear localized modes and soliton dynamics supported by the PT-symmetric couplers or lattices under a variety of settings [289–293].

6. Concluding remarks

The field of optical spatial solitons really began back in the 1970s and reached some maturity in the 1990s, and is still at the frontier of nonlinear optics and photonics. Although the formation mechanism and fundamental properties of spatial solitons under different nonlinearities have been studied extensively, much remains to be uncovered, especially with new developments in nonlinear synthetic materials including photonic bandgap materials, metamaterials, soft condensed matter and PT-symmetric materials. Judging from the current level of research activity, we believe that the area of spatial solitons will continue to flourish for many years to come.

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