Anomalous interactions of spatial gap solitons in optically induced photonic lattices

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We demonstrate coherent interactions between spatial gap solitons in optically induced photonic lattices. Because of the "staggered" phase structures, two in-phase (out-of-phase) bright gap solitons can repel (attract) each other at close proximity, in contrast to soliton interaction in homogeneous media. A reversal of energy transfer direction and a transition between attractive and repulsive interaction forces can be obtained solely by changing the initial soliton separation relative to the lattice spacing. © 2011 Optical Society of America

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Interaction between optical spatial solitons has been studied intensively for decades due to their particlelike behaviors and promise for controlling light with light [1]. In homogeneous nonlinear media, it is well known that interaction between two coherent solitons depends on their initial phase relation. In particular, two initially parallel in-phase (out-of-phase) solitons at no relative transverse velocity simply attract (repel) each other, while for other phase relations, energy transfer occurs between the interacting solitons [2,3]. In the past decade, spatial discrete solitons in photonic lattices attracted a lot of attention [4–6], with many fascinating soliton phenomena uncovered as a result of interplay between nonlinearity and bandgap structures. Thus far, investigation on the interaction between two fundamental lattice solitons has reached a similar conclusion as that in homogeneous media, that is, in-phase solitons attract each other, while out-of-phase solitons repel [7–9]. On the other hand, interaction between two spatial gap solitons has not received much attention. Under a self-defocusing nonlinearity, it was shown that soliton fusion of two in-phase gap solitons occurs at low power levels [10], similar to the interaction of semi-infinite gap solitons under self-focusing nonlinearity. In this Letter, we demonstrate anomalous interactions of mutually coherent gap solitons in optically induced photonic lattices. We show that two in-phase (out-of-phase) gap solitons can repel (attract) each other during soliton collision, and a reversal of energy transfer direction and a transition between attractive and repulsive interaction forces can be obtained solely by changing the initial soliton separation. These results may be applicable to gap soliton interaction in other nonlinear discrete systems.

We consider two one-dimensional (1D) gap solitons interacting in the optically induced lattice in a photorefractive crystal [as shown in Fig. 1(a)]. The nonlinear propagation of two soliton beams is governed by the normalized nonlinear Schrödinger equation under saturable nonlinearity:

$$\left(\frac{\partial}{\partial z}-\frac{\mathrm{i}}{2}\frac{\partial^2}{\partial x^2}\right)B_i(x,z)=\mathrm{i}E_0\left(\frac{I_l+I_p}{1+I_l+I_p}\right)B_i(x,z),\quad i=1,2, \tag{1}$$

where $B_i(x,z)$ is the complex amplitude of one of the probe beams, E_0 is the external bias field, $I_l=I_{l0}\cos^2(\pi x/d)$ is the intensity of the lattice-inducing beam with the intensity peak I_{l0} and period d, and $I_p=|\sum B_i|^2$ denotes the total intensity of the two coherent beams. The dimensionless parameters (x, y, z, and E_0) are related to the physical parameters (x', y', z', E_0') by the expressions $(x,y)=(k\alpha)^{1/2}(x',y')$, $z=\alpha z'$, and $E_0=E_0'/E_n$, where $\alpha=kn_e{}^2\gamma_{33}E_n$. Here k is the wave number of light in the crystal, n_e is the extraordinarily polarized index of refraction, γ_{33} is the effective element of the electro-optic tensor, and E_n is the normalized constant of the bias field. For a single probe beam B_1 (i.e., $I_p=|\mathrm{B}_1|^2$, the gap soliton solutions of Eq. (1) can be found in the form $\mathrm{B}_1(x,z)=b(x)\exp(i\beta z)$, where β is

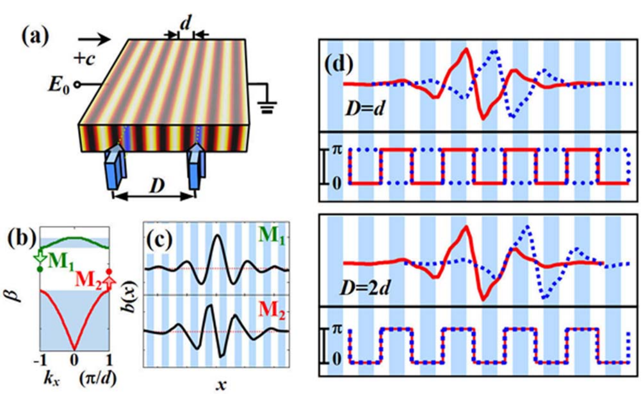

Fig. 1. (Color online) (a) Schematic of gap soliton interactions in an optically induced photonic lattice. (b) Bandgap structure and (c) first- and second-band gap solitons with the low-index regions shaded. (d) Overlapping of two second-band gap solitons at different separations.

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the propagation constant and $b(x)$ is the real envelope. Two typical gap soliton solutions found in the first Bragg reflection gap [Fig. 1(b)] are illustrated in Fig. 1(c), corresponding to the first-band (top) and second-band (bottom) gap solitons bifurcated from the high-symmetry points M_1 and M_2 under opposite nonlinearities. Because gap solitons have characteristic "staggered" phase structures [5,6], we expect that the dynamics of coherent interaction of two gap solitons depends not only on their initial phase relation but also on their initial separation D. For example, let us look at two mutually in-phase gap solitons (notice that each has a "staggered" phase structure) bifurcated from point M_2, as shown in Fig. 1(d). When the two gap solitons are separated by an odd number of d [see the top panel in Fig. 1(d)], they will overlap with an opposite phase and interfere destructively. Likewise, when they are separated by an even number of d [see the bottom panel in Fig. 1(d)], they will be in phase and interfere constructively. Because dynamic interaction of two mutually coherent solitons is determined by the interference between their evanescent tails [1–3], different interacting forces can be triggered depending on their initial separation relative to lattice spacing, even if they have fixed initial phase relation. In particular, two in-phase gap solitons would repel each other should they be separated by an odd number of lattice period d.

To show such "anomalous" behavior of interaction between gap solitons, let us first perform numerical simulations for two identical gap solitons bifurcated from point M_2. The two solitons (say B_1 and B_2) are simultaneously normally launched into different lattice sites with an initial phase difference $\Delta\varphi$ [Fig. 1(a)]. In order to make the interaction dynamics more perceptible, the gap solitons are chosen to have wide spatial distributions (less localized) to ensure larger superposition regions. Here the parameters used are $d = 5$, $I_{l0} = 0.5$, $I_{\max} = |B_1|^2{}_{\max} = |B_2|^2{}_{\max} = 0.05$, and $E_0 = 2$. Corresponding beam propagation method (BPM) simulations of coherent soliton interaction at normalized propagation distance $z = 200$ are illustrated in Fig. 2, where the soliton separation D for 2(a)–2(d) is $3d$, $4d$, $5d$, and $6d$, respectively, for different phase differences $\Delta\varphi$. It can be seen clearly that under the self-focusing nonlinearity, the in-phase gap solitons separated by an odd number of lattice spacing repel each other [Figs. 2(a1) and 2(c1)], although the two solitons attract (behave "normally") when they are separated by an even number of lattice spacing [Figs. 2(b1) and 2(d1)]. Likewise, the interaction between gap solitons with $\Delta\varphi = \pi/2$ and π also leads to unusual dynamics. For out-of-phase gap solitons, they attract (or repel) each other when separated by an odd (or even) number of d [Figs. 2(a3)–(d3)]. Especially, when $\Delta\varphi = \pi/2$, energy transfer between two solitons occurs during interaction, and the direction of energy transfer is switched merely by altering D [see Figs. 2(a2)–(d2)]. The separation-dependent anomalous interaction behaviors are unique to "staggered" gap solitons. Note that the underlying physics of such anomalous interaction behaviors is much different from that of the similar separation-dependent phenomena observed in the linear regime [11]. In the linear case, the $\pi/2$ phase shift of the light amplitudes between the adjacent waveguides due to evanescent coupling is the key for the alternating behavior between

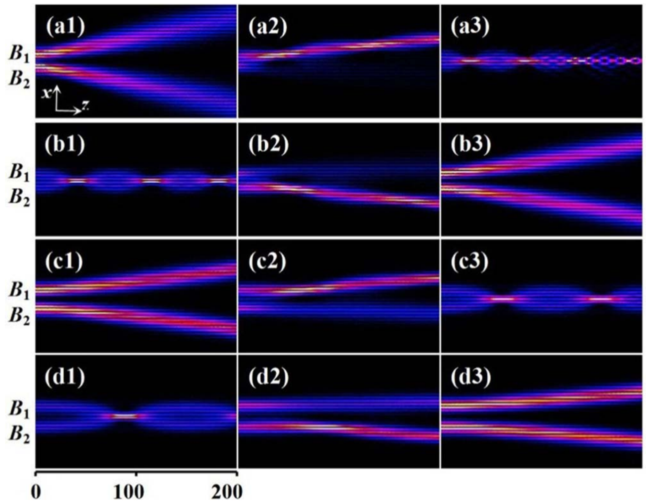

Fig. 2. (Color online) BPM simulations of coherent interactions of gap solitons at different separations D and phase differences $\Delta\varphi$. (a)–(d) $D = 3d$, $4d$, $5d$, and $6d$. Left to right, $\Delta\varphi = 0$, $\pi/2$, and π.

quasi-incoherence and coherence at different separations [11]. While for the gap solitons, which are nonlinear localized Bloch modes, the phase difference between the light amplitudes at two adjacent waveguides is always π; therefore, the interaction between two gap solitons is always coherent.

To demonstrate the anomalous interaction between gap solitons, we perform a series of experiments in 1D photonic lattices optically induced in a biased SBN:60 ($5\,\mathrm{mm} \times 10\,\mathrm{mm} \times 5\,\mathrm{mm}$) crystal. Considering that a gap soliton bifurcated from point M_1 can be easily excited by a single Gaussian beam under self-defocusing nonlinearity [6], we use off-site excitation of two in-phase Gaussian beams to study first-band gap soliton interactions. The experimental setup is illustrated in Fig. 3. The lattices ($11\,\mu$m period) are established by sending an ordinarily polarized partially coherent beam ($\lambda = 488\,\mathrm{nm}$) through an amplitude mask and then to the negatively biased crystal. Two extraordinarily polarized probe beams (B_1 and B_2) are generated with a triangular interferometer, where a single probe beam splits into two beams. To separate these two beams at crystal input,

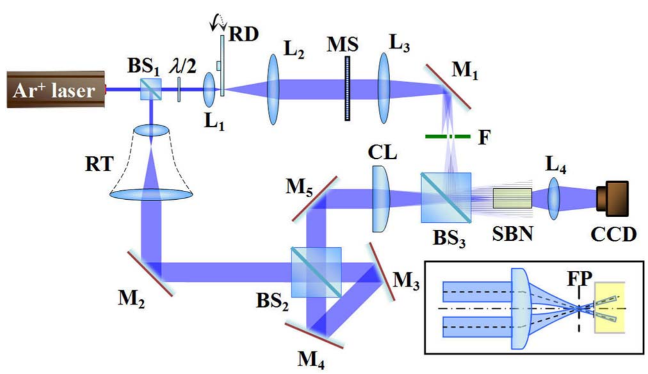

Fig. 3. (Color online) Experimental setup for observing coherent interactions between gap solitons: SBN, strontium barium niobate; BS, beam splitter; $\lambda/2$, half-wave plate; RT, reversed telescope; RD, rotating diffuser; L, lens; MS, amplitude mask; M, mirror; F, Fourier-plane filter; CL, cylindrical lens; and FP, focal plane.

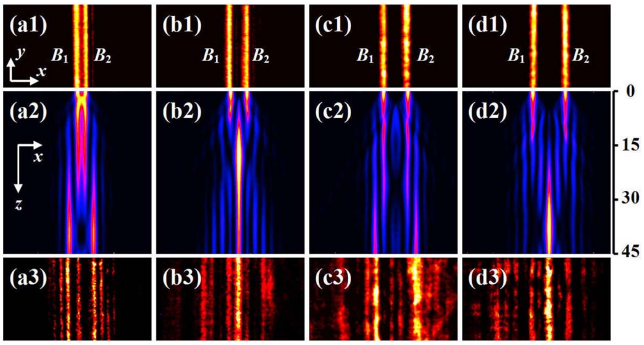

Fig. 4. (Color online) Experimental observation (top panels, input; bottom panels, output) and numerical simulation (middle panels) of in-phase gap solitons interacting in an optically induced photonic lattice. The initial beam separation in (a)–(d) is $D = d$, $2d$, $3d$, and $4d$, respectively.

their minimum beam waists are relocated behind the focal plane, and their spacing are adjusted by transversely shifting only one of beams through beam splitter BS_2 while keeping the same initial phase. At first, only one of the probe beams (B_1 or B_2) with $9\,\mu$m FWHM is launched into the crystal. The intensity ratio between the probe beam and the lattice-inducing beam is about 1:5. By applying a voltage of -900 V, a gap soliton forms whose intensity profile covers quite a few waveguide channels. Then, both probe beams (B_1 and B_2) are launched simultaneously with different separations, while in-phase and other experimental conditions remain the same. The experimental results are shown in Fig. 4, where (a)–(d) correspond to beam separations $D = 11$, 22, 33, and $44\,\mu$m, respectively. The experimental input and nonlinear output are shown in the top and bottom panels, while the middle panels show the corresponding beam evolution from numerical simulation with normalized parameters of $d = 4.1$, $I_{l0} = 1$, $I_{\max} = 0.2$, $E_0 = -1.8$, and $z = 45$ according to our experimental conditions. From these experimental results, it can be seen that even though the two beams at the input facet are kept in phase for different separations, the outcome of the nonlinear interaction is dramatically different. As shown in Figs. 4(a) and 4(c), at the separation of an odd number of the lattice period, the two gap solitons repel each other, representing anomalous interactions. Only when the separation is an even number of the lattice period, do the two in-phase solitons attract each other [see Figs. 4(b) and 4(d)], similar to the interaction of semi-infinite-gap discrete solitons [7]. Clearly, our experimental results agree well with the numerical simulation.

We have also performed a series of experiments and numerical simulations under other nonlinear conditions. For instance, when the nonlinearity is not strong enough to support gap solitons, similar interacting behavior, as shown in Fig. 4 is also observed, provided that the nonlinear propagation leads to "staggered" phase structures. Numerically, we found that the gap solitons residing in higher gaps undergo similar anomalous interaction due to their staggered phase structure, but the soliton separation for attraction or repulsion is different. For instance, the gap solitons residing in the second photonic gap interact anomalously (i.e., in-phase solitons repel each other) when the initial separation is an odd number of a half-lattice period $d/2$ instead of a whole lattice period d. Intuitively, two mutually in-phase gap solitons, regardless of which gap they reside in, can repel (attract) each other as long as their "staggered" phase structure has an antiphase (in-phase) overlapping, as illustrated in Fig. 1(d).

To summarize, we have studied numerically and experimentally the interaction of spatial gap solitons in photonic lattices. Because of the staggered phase structures of gap solitons, the interaction dynamics are determined by both the initial relative phase and the initial separation of the solitons. We expect similar phenomena of gap soliton interactions to occur in other nonlinear discrete systems.

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